Problem Solving in Mathematics

An addition to a Mathematics Teacher's Toolkit

Bolton Learning Alliance

Bolton Council

Contents

Introduction

This booklet contains several ways of helping pupils to approach problem solving in KS3 mathematics lessons. It is limited in its scope and only explores some of the available material. In the appendix you will find references to other approaches.

All mathematics questions are about solving problems. In this booklet we are specifically looking at questions that are provided in the form of words and consider strategies which might help pupils translate the words into mathematics.

As thinking in mathematics is complex, no one solution will work for all pupils. We are therefore describing a variety of methods for you to explore and hopefully you will then have a better toolkit to rely on in lessons. We are certain that you will have alternative methods. Please be willing to share them by bringing them to the Bolton Mathematics Hub meetings.

We are grateful to the following people for their ideas which are included in this booklet: Eric Tuckley, Phil Meek (Westhoughton High School), John Gunn (Ladybridge High School), Rebecca Reevell (Mount St Joseph Business and Enterprise College).

Reading the problem

If pupils cannot 'read for comprehension' they cannot solve problems. As teachers of mathematics we are just as much teachers of reading as we are teachers of mathematical processing. Pupils need to be able to read mathematics when it is expressed in its abstract form and when it is expressed as word problems.

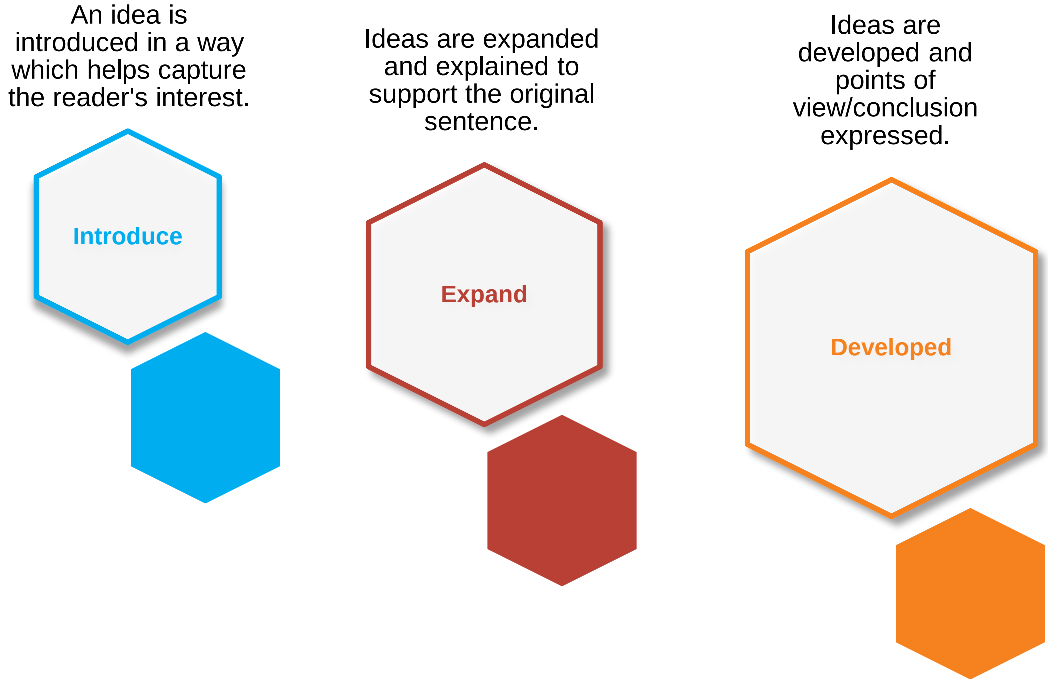
To improve a pupil's reading comprehension think about using the following strategies:

Read Slowly	Read the problem slowly, word by word. Teach pupils to slow down: pupils like to rush through reading and end up with mental inaccuracies that affect understanding.
Underline	A technique to slow pupils' reading speed down is to get them to underline the most important pieces of information.
Concentrate	When reading, pupils need to concentrate on separating complex sentences into phrases.
Read out Loud	Reading out loud improves the quality of a pupil's understanding and helps pupils to focus.
Re-Reading	Re-reading the problem is a simple strategy that helps develop understanding. This helps pupils to work out which information and numbers relate to the question being posed.
Decide	After reading the problem – decide which aspect of mathematics the question relates to and recall rules and processes within that branch.
Research	Pupils should research any technical words that they don't know.

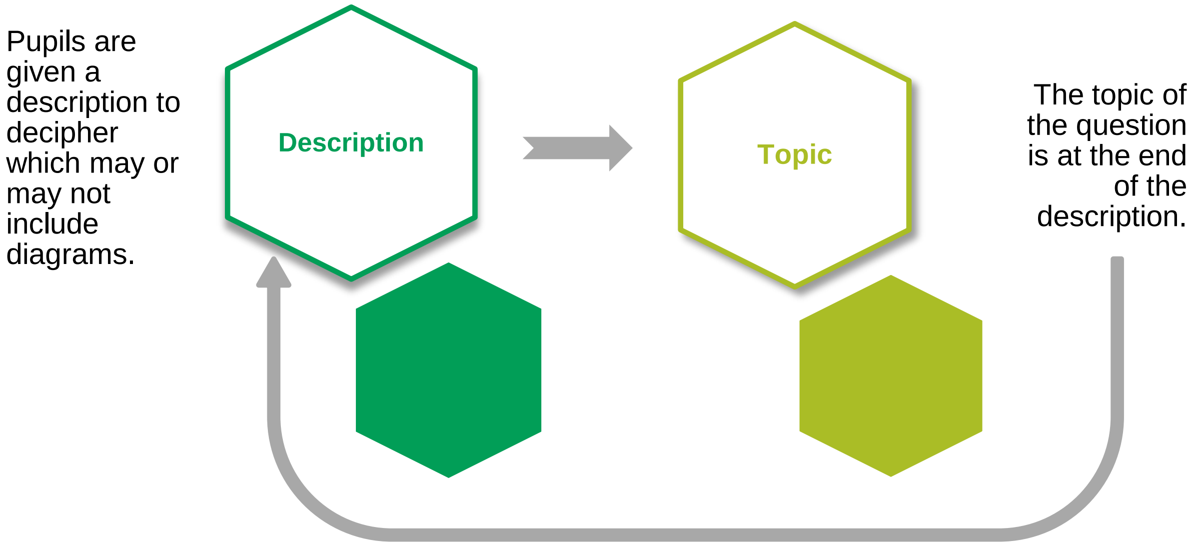
Immerse	Immerse pupils in the vocabulary of problem solving. Rather than concentrate on single words, teach the common phrase that the words are used to form. Use mathematical dictionaries, books, glossaries.
Practice	During normal classwork, practice getting pupils to explain the steps that they have taken to solve any mathematics using full sentences and proper mathematical vocabulary. Don't accept low level explanations. Make then build proper sentence answers that explain fully their reasons and choices. Get pupils to help each other to build good answers.
Be wary	Be wary of one pupil being able to explain a process. It does not mean that the rest of the class understand.
Record	Record them presenting answers on their own phones then ask them to practice improving the way they explained as part of their homework.
Modelling Thinking	Teachers modelling their thinking out loud is valuable, particularly for disadvantaged pupils.
Don't Explain	Don't explain the text to them – make them work it out in response to your questioning.

How is reading in mathematics is different to reading in English?

English explanations are often written in the following form:



In mathematics questions the format is more often like this:



It is because the topic is at the end of the question that re-reading the question helps pupils to understand it better.

Processing diagrams and relating them to the text can take a lot of mental effort. Annotating diagrams with information from the text can help clarify thinking.

Also, in mathematics the sentence structure is often complex therefore asking the pupils to reword it helps.

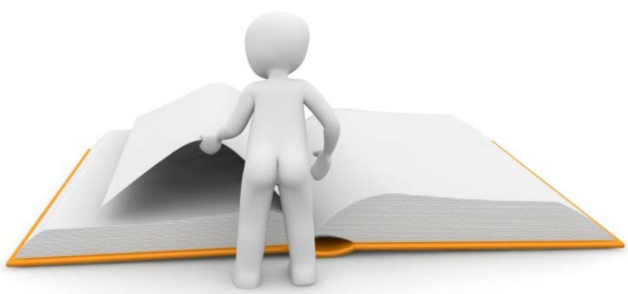
Sweat the small stuff:

Vocabulary Words and Their Meanings

Many small words have specific meanings in mathematics. Teaching these can help pupils to make sense of the questions. 'Of' and 'off' are classic examples of words which cause confusion. But there are many others. Time spent teaching the specific meaning of these words will help pupils' solve problems more effectively. How about having a literacy month with year 7 where learning the meanings and usage of these words are a focus of every mathematics lesson?

Word	Main meanings
The	One specific thing
Is	Equals
A	Any one thing
Are	Equals
Can	Able to
On	On top of and under
Page	One sheet in a book
Who	The question is asking about someone
Find	Work out or calculate
One	An entity / idea that stands for more than 0 and less than 2
Ones	Position; in a number, the numeral to the far right (Think HTU)
Unit	An item; a whole number between and including 0 and 9
Units	In a number, the numeral to the far right (Think HTU)
Ten	A number / idea that stands for more than 9 and less than 11
Tens	Position; in a figure, the numeral to the left of the ones (Think HTU)
Hundred	Idea that stands for more than 99 and less than 101
Hundreds	Position; in a figure, the numeral to the left of the tens (Think HTU)
And	Something more, do both
Or	Either this or that but not both
Number	Abstract idea / concept used for counting and measuring
Numeral	Symbol used to write a number
How	Question word asking for the step or steps used in calculating
Many	Amount, contrasted to few

Word	Main meanings
How Many	Question asked for the number of something
Few	A small number of …
Less	A smaller amount of; not as much; minus
What	Question asking for things as opposed to persons
You	Contrast to me, statement directed to you
Your	Contrast to mine, shows ownership
We	Group including self, usually the subject of the sentence
It	Contrast to he / she, in maths it refers to a problem or thing
Look	Command to use eyes and allow brain to react
Write	Put pencil in hand and make a mark, symbol, etc. Not ‘write’ as in cursive
Each	Every single one
This	Specific item / idea in close location
That	Contrast to this, specific item / idea but not in close location; (multiple other meanings)
Set	Group of things with something in common
Us	Group including self, usually the object of the sentence
There	Contrast to here, not in close location
Which	Starts a question that implies a choice
Do	Work out or calculate
Same	Alike, not different; equal in meaning
Exercises	A group of problems (not physical activities)
These	Contrast to those, more than one in a close location
First	Contrast to then, usually means spatial, e.g. first in line; in maths has to do with time, e.g.. Do this first
Have	Contrast to have not or had, hold in one’s possession. Can be past, present or future tense in meaning.
Here	Contrast to there, here is in a close location
Times	Multiply; in “How many times” it may also mean the number of trials or performances.
Has	Possession of, singular form - present tense.
All	Everything or everyone
Equals	Is, are, or the same amount on both sides



The HATS approach

H	Highlight key words, information
A	Annotate formulae linked to key words, lingo, definitions, sketches
T	What is the Topic in the question about?
S	Steps and marks: put in the steps, have you done enough to get the marks?

Rebecca Reevell

How to approach a problem solving question

(photocopiable sheet available in the appendices)

The problem:

Step 1: Understand the problem

What are you trying to find or do? Can you state the problem in your own words? What information can you obtain from the problem? What information, if any, is missing or not needed?

Step 2: Devise a plan

Look for a pattern. Use a flow diagram. Use x for the unknown. Consider similar problems. Draw a diagram. Make a table. Use simpler numbers. List the possibilities. Guess and check.

Step 3: Carry out the plan

Keep an accurate record as you go. Check each step as you go. Are you keeping to your plan? Found a solution?

Step 4: Review your solution

Interpret your solution. Check your final answer. Check for other answers. Review your strategy. Does your solution seem reasonable?

John Gunn

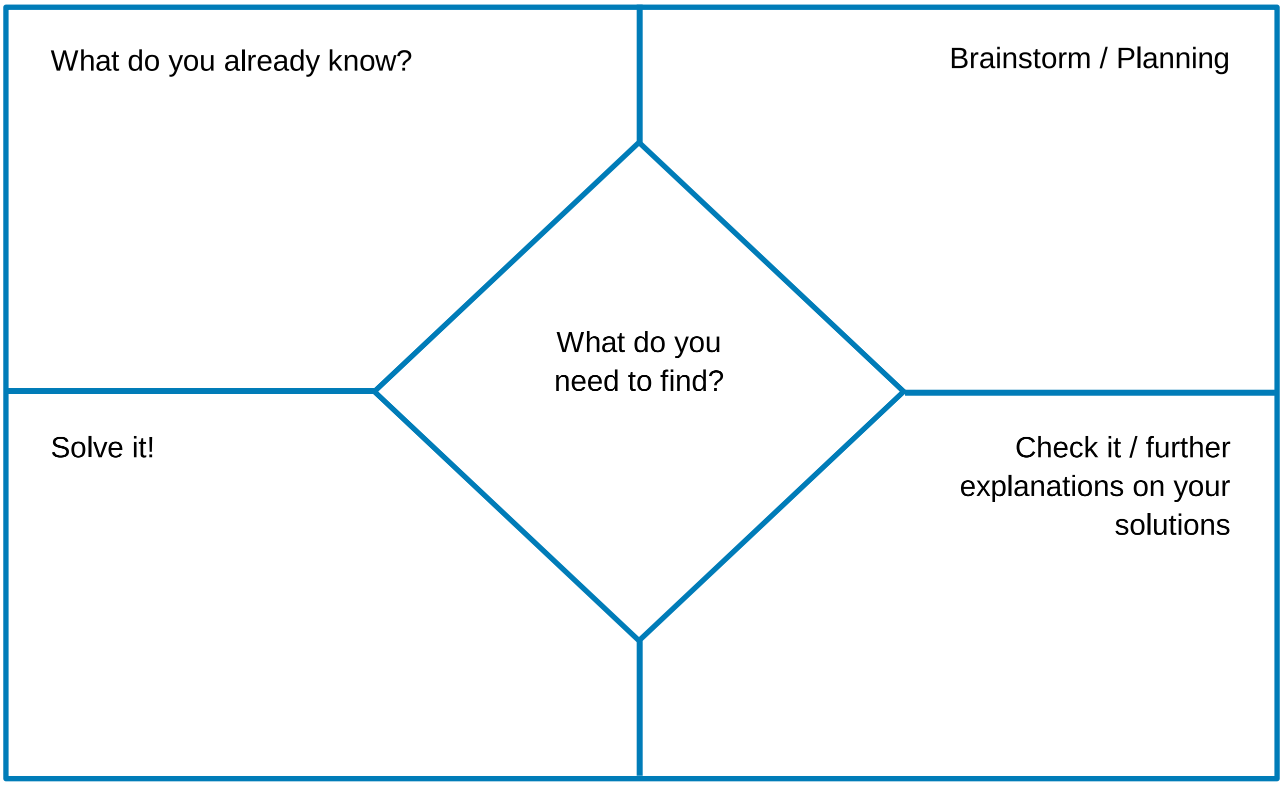
4 Corners and a Diamond

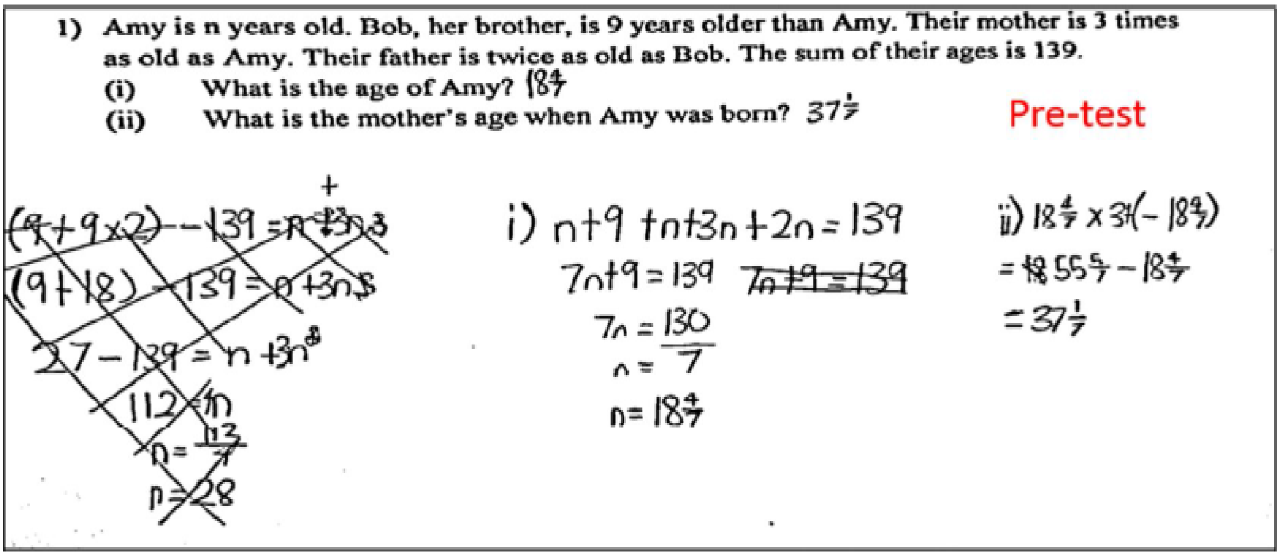
The four corners and a diamond graphic organiser has five areas:

1. What do you need to find?
2. What do you already know?
3. Brainstorm possible ways to solve this problem.
4. Try your ways here. Solve it.
5. Check it. What further explanations do you need to give?

Then a question to ask at the end:

What mathematics did you learn by working through this problem?





1) Amy is n years old. Bob, her brother, is 9 years older than Amy. Their mother is 3 times as old as Amy. Their father is twice as old as Bob. The sum of their ages is 139.

(i) What is the age of Amy? $18\frac{4}{7}$

(ii) What is the mother's age when Amy was born? $37\frac{1}{7}$

Pre-test

i) $n+9+n+3n+2n=139$

$7n+9=139$ ~~$7n+9=139$~~

$7n=130$

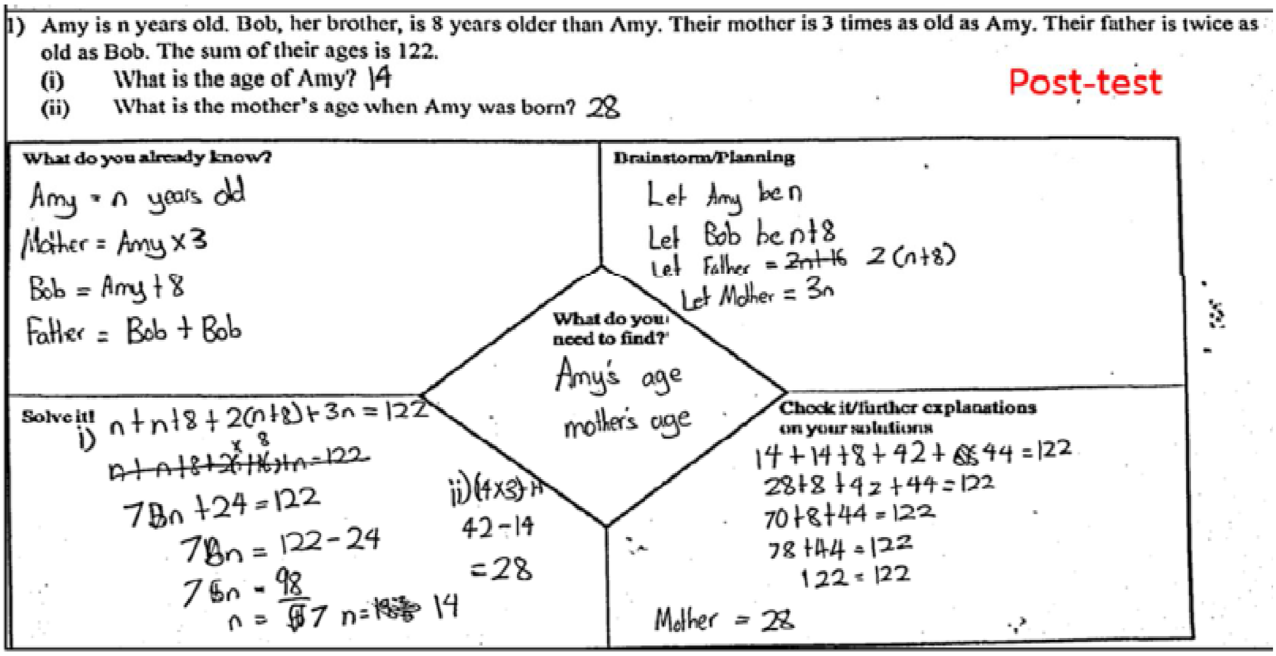
$n=\frac{130}{7}$

$n=18\frac{4}{7}$

ii) $18\frac{4}{7}\times 3(-18\frac{4}{7})$

$=55\frac{5}{7}-18\frac{4}{7}$

$=37\frac{1}{7}$



1) Amy is n years old. Bob, her brother, is 8 years older than Amy. Their mother is 3 times as old as Amy. Their father is twice as old as Bob. The sum of their ages is 122.

(i) What is the age of Amy? 14

(ii) What is the mother's age when Amy was born? 28

Post-test

What do you already know?

Amy = n years old

Mother = Amy x 3

Bob = Amy + 8

Father = Bob + Bob

Brainstorm/Planning

Let Amy be n

Let Bob be n+8

Let Father = 2n+16 2(n+8)

Let Mother = 3n

What do you need to find?

Amy's age

mother's age

Solve it!

i) $n+n+8+2(n+8)+3n=122$

$7n+24=122$

$7n=122-24$

$7n=98$

$n=\frac{98}{7}$ $n=14$

ii) (14x3)-14

42-14

=28

Check it/further explanations on your solutions

14+14+8+42+44=122

28+8+42+44=122

70+8+44=122

78+44=122

122=122

Mother = 28

Having been taught how to use the 4 corners and a diamond organiser, pupils then approach the problem like this:

From: Khoo, Shahrill, Yusof, Chua, & Roslan, Graphic Organizer in Action Journal on Mathematics Education: Volume 7, No. 2, July 2016, pp. 83-90

Comment:

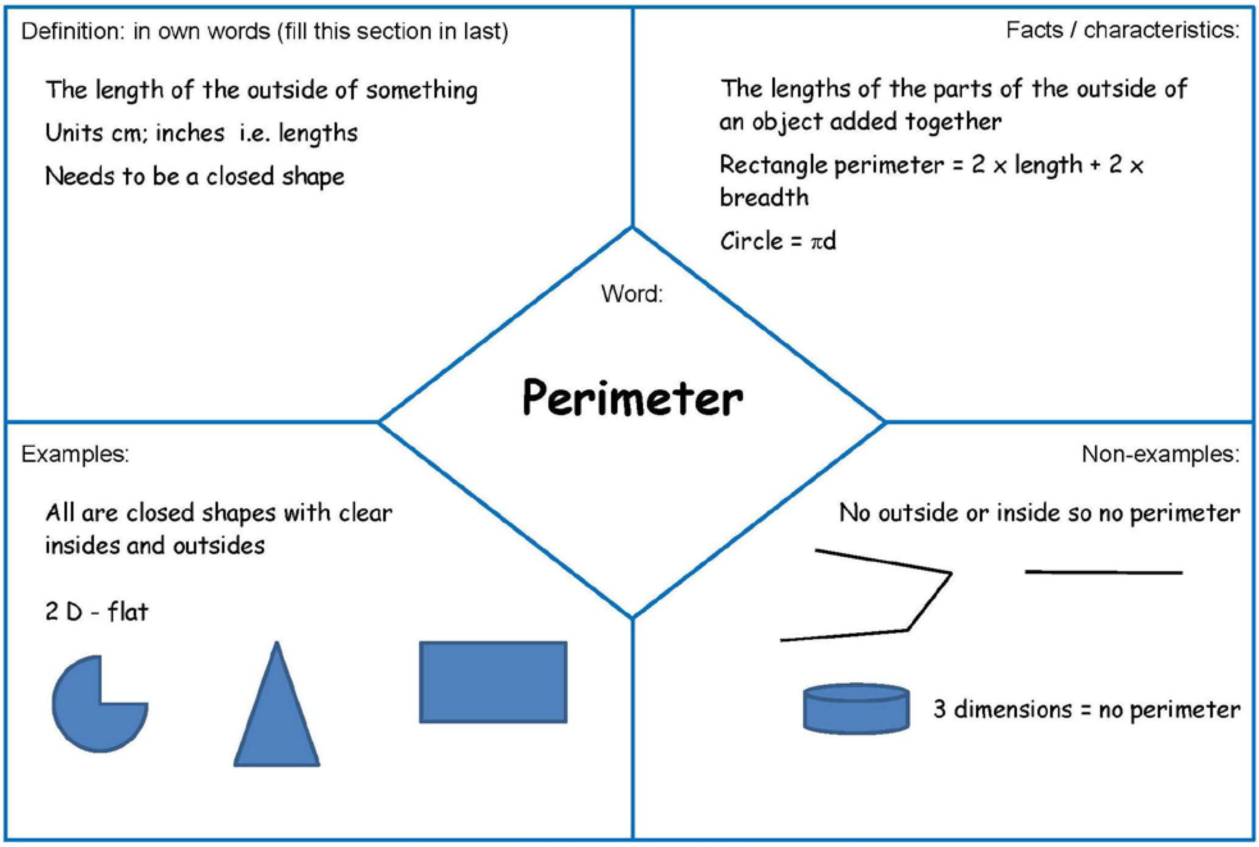
Although this organiser can help the pupils to understand the problem better, it can be time consuming to complete. The ‘What do you already know?’ section can be sentences or diagrams.

An aside:

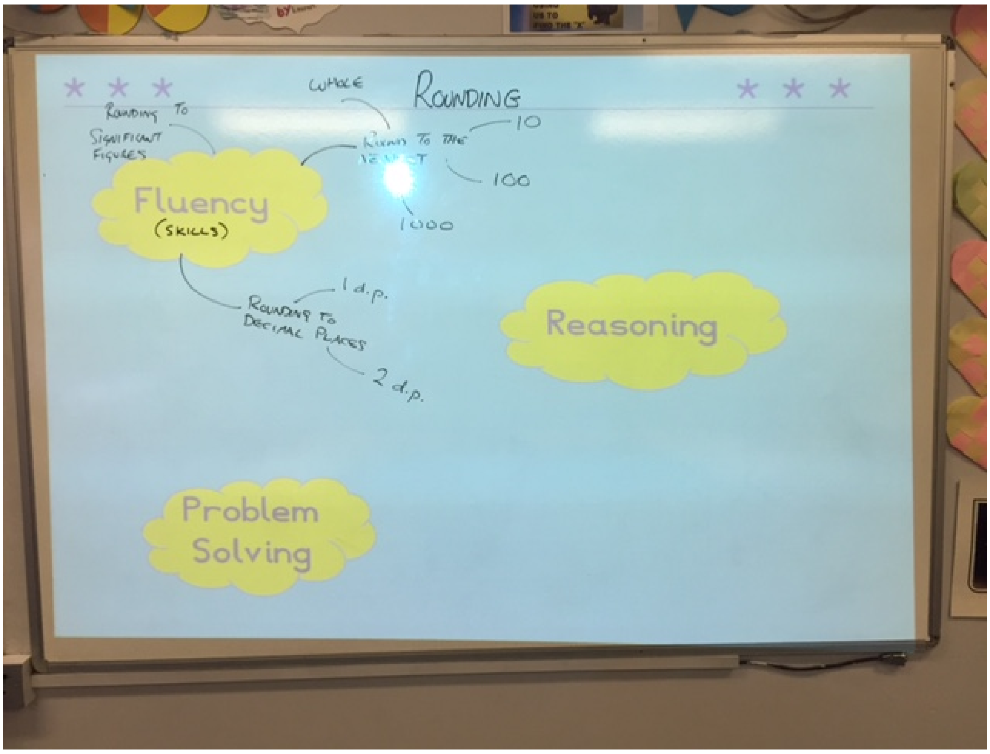
A similar format is used in Frayer diagrams which help pupils to organise their thoughts regarding what they know about a topic.

It is worth trying in this order:

- Word
- Facts / characteristics
- Examples
- Non-examples
- Definition



Fluency, Reasoning and Problem Solving



The picture shows the class contribution after starting a session on ‘rounding’. We returned to this picture (which was also in their exercise books) once we had done some reasoning e.g. how can rounding help us ‘estimate’ the answer to a calculation. Students completed more of the clouds as we progressed. In this session the ‘estimating’ task was shown as part of the ‘reasoning’ cloud, however next time we visit the topic of ‘estimating’ it will be part of the ‘fluency’ cloud.

Students now have a collection of cloud diagrams in their books with information about how they have become fluent, reasoned and solved problems. Many have told me they enjoy this part of the lesson as they can quantify what they have learned (new skills) and understand where they have reasoned (applied fluency skills) to solve more complex problems. I will continue to trial this.

Phil Meek

Singapore mathematics problem solving

Singapore mathematics inserts a vital step into the process of moving pupils from concrete thinking into abstract thinking. This step uses pictures and images to sort out the proportions and relationships described in a text. The process of constructing the picture helps pupils to visualise the known relationships from the description and thereby work out missing, unknown information.

The thinking finally becomes abstract when the pupil is able to use just numbers and symbols. It is particularly useful as an approach when working with whole numbers, fractions, proportions, ratios and percentages.

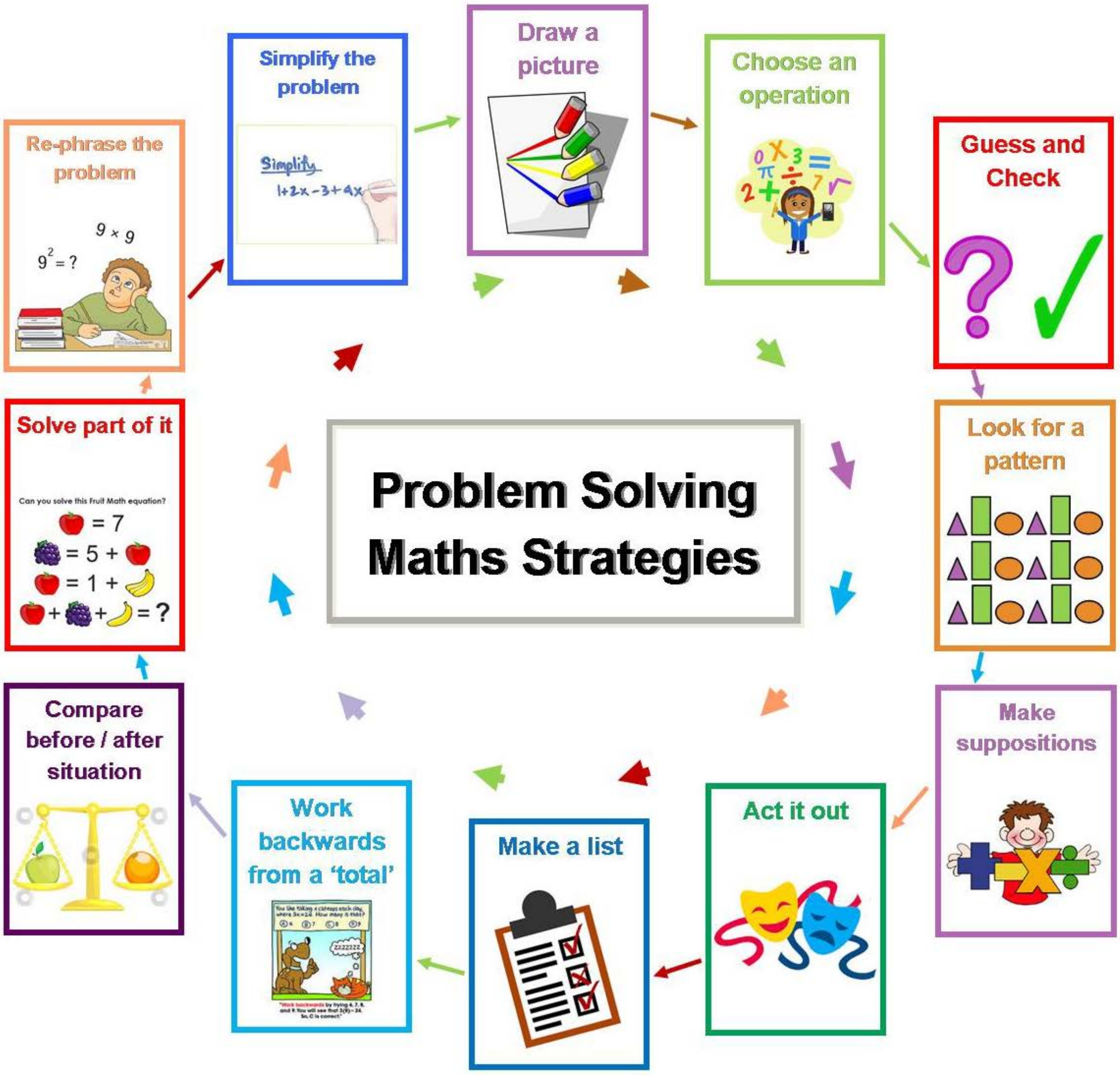
Pupils find the processes easier if they know their number bonds (both addition and subtraction) and the multiplicative relationships between whole numbers.

In addition to the use of pictures to represent relationships in problems, the process is seen as a series of steps:

1. Understand the problem
2. Plan what to do
3. Work out the answer
4. Check the answer

The ‘plan what to do’ section involves pupils making choices about which strategy to use.

These are some of the strategies taught in Singapore mathematics :



1. Analysing parts and wholes and comparing

Singapore mathematics often uses the bar method as the basis to draw the diagram. It is adaptable to many situations and can show whole and parts in relation to each other and therefore help pupils to understand what information they have and what they need to find.

There are two basic methods used which we will explore below:

- Where the full line represents to whole which can be broken into parts.
- Where several lines are drawn for comparison reasons.

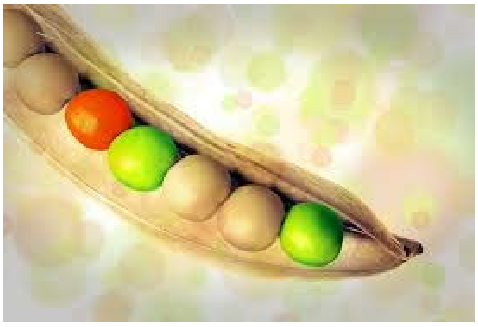
a. Analysing parts and the whole

- Pupils are taught to read a problem and identify which values and which sections of the text refer to the total **and** which values and sections relate to parts of the whole.
- The pupils are then encouraged to see if the parts can be easily be broken down into a basic chunk that has been used to build the 'whole'. Missing sections can then be identified and calculated.

Example:

On a family farm which grows peas there are 2 women workers and 5 men workers. In 3 hours, they managed to pick 109 kilograms of peas between them. If each man managed to pick 5 kilos of peas more than each woman and each kilo is sold for £3.50, how much money would each man make?

(How would the complexity of this question change if the pupils were asked to find out how much money the men make? – small changes in wording can significantly affect the problem to be solved.)



109 kg	–	25 kg	=	84 kg
84 kg	÷	7 people	=	12 kg each

Men picked

12kg	+	5 kg	=	17kg
17 kg	x	£3.50	=	£59.50 made per man

£59.50 per man x 5 men = £297.50

b. Comparing

This is similar to the previous method only this time two or more bars are drawn so that comparisions can be easily made. Bar diagrams are particularly useful for this as they easily allow visual comparisions. Bars are more effective and useful than circles as they are more adaptable to different circumstances.

Example

The ratio of the area of a triangle to the area of a rectangle is 3 : 5.

The difference between the areas is 96 cm^2.

What is the height of the triangle if its base is 2 times its height?

Triangle area				$\overbrace{\qquad\qquad}^{96\ cm^2}$	
Rectangle area					

Each block is called a 'unit'

From the diagram

2 units → 96 cm^2

1 unit → 48 cm^2

Therefore the area of the triangle is:

3 units → 4 x 48 = 144 cm^2

Consider the triangle:

The area of a triangle is also

½ base x height = 144 cm^2

base x height = 144 cm^2 x 2

= 288 cm^2

If the base is 2 times the height this can also be written:

2 x height x height = 288 cm^2

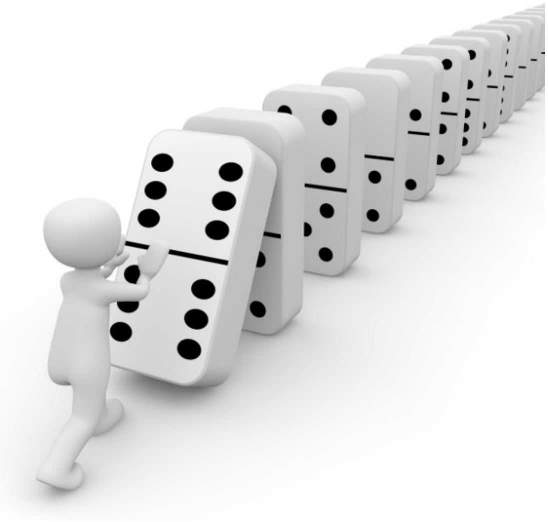
Therefore:

height x height = 144 cm^2

Height = 12 cm

(It is also worth noting that, at this stage in the Singapore maths system, the units are always written beside the number at every stage rather than added at the end of the calculation as this help pupils keep track of what they are looking for).

2. Patterns in sequences



This is a process of writing out a sequence of numbers or letters and looking for a pattern in the sequence. It requires pupils to be very familiar with number bonds and multiplication tables. They need to be confident playing with numbers and looking for patterns.

3. Deduction or induction – which one when?

These two methods require different mental processing. At key stage 3 think of it like this:

a. General statements → Special conclusion → Deduction

Deduction is drawing a conclusion from something known or assumed. Logic tables and solving equations are examples of deduction.

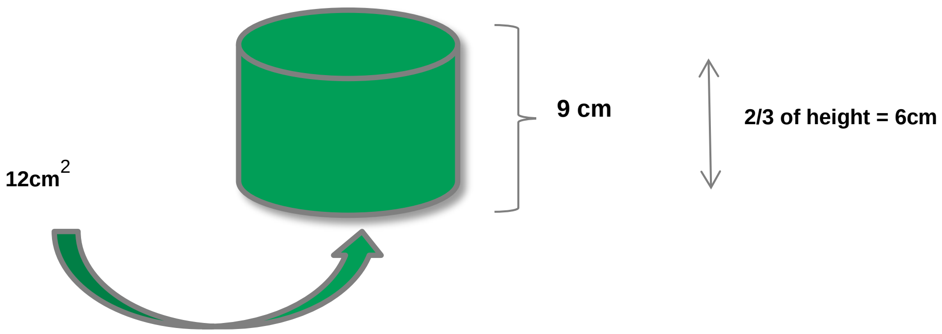
b. Specific cases → General conclusion → Induction

Prove something true for n=1. Assume it is true for any n and then prove it true for n+1. (Prove means to use deductive logic). Effectively what happened is that truth for n=1 implies truth for n=2 which implies truth for n=3, etc. (Be careful induction in science is very different)

4. Draw a diagram (more visualisation)

A cylinder is closed at one end and open at the other. What volume of liquid will it hold if it has a cross sectional area of 12 cm^2 and can only be filled two thirds full. Its height is 9 cm.

Taking the information from the text and drawing a diagram to match it makes the solution much clearer.



Reduce to one and multiply up

This is an easily taught method which has many applications.

Typical problems would be ‘if 5 kilos of potatoes cost £3, how much will 8 kilos cost?’ and ‘if it takes 3 days for 5 people to dig a ditch, how many people are needed to finish the job in one day?’

Reduce to one and then multiply up.

a. The potatoes question is a direct proportion question such that an increase in one quantity causes a corresponding increase in the other.

5 kilos of potatoes cost £3

1 kilo of potatoes cost £3 ÷ 5, i.e. £0.60

8 kilos of potatoes cost £0.60 x 8

Answer £4.80

b. The people digging a ditch is a problem using inverse variation, i.e. a relationship between two variables such that an increase in one causes a corresponding decrease in the other, or a decrease in one causes a corresponding increase in the other.

For example, the more people who are digging, the shorter the time taken to dig the ditch; conversely, the fewer people who are working, the longer the time taken to dig the ditch; therefore, the number of people digging the ditch is inversely proportional to the number of days needed.

This is often a more difficult concept, with pupils asking which <u>one</u> do I reduce it to?

It takes 3 days for 5 people to dig a ditch.

It takes 3 x 5 days for 1 person to dig a ditch.

It takes 15 days for 1 person to dig the ditch.

Therefore it would need 15 people to dig the ditch in one day.

Specialising, generalising and the value of getting stuck

Mathematical problem solving can be looked upon as a process of trying out an idea with actual numbers followed by looking for patterns which allow us to come up with a more general solution that can be applied in every circumstance.

In technical terms this is known as:

- Specialising
- Generalising

Specialising is to explore the relationships in a problem by using numbers (numbers or diagrams **not** algebraic symbols). The purpose of doing at this stage is twofold:

- to help with understanding the meaning of the problem and
- to look for patterns.

These are two completely different mental processes and pupils should be encouraged to recognise the different stages within specialising. This is metacognition and helps with long term memory and applying the processes again in new circumstances.

Pupils should be encouraged to be systematic in the trial of different numbers.

Generalising is a process by which the pattern is sensed and then expressed. The answer is refined by testing with specific examples /numbers to see if the proposed answer holds true (this is a further use of specialising). This testing is an important phase of the mental processing. Pupils should be encouraged to try wide ranging examples.

Some pupils will be reluctant to guess the pattern – others will be 'more hasty' in the assumptions they make and will need to be taught to be more systematic.

Getting stuck

Getting stuck is a normal part of the thinking process. Articulating why they are 'stuck' is useful to the pupils. You experience this whenever you are 'one to one' teaching – getting an individual to explain what the problem is can help to release the solution. As you cannot be there every time they get stuck then encourage them to write:

- What don't they understand?
- What don't they know how to do?
- What needs more explanation?
- Are they being systematic in their thinking or just having 'wild' guesses? How can they be more systematic?

Sometimes the getting stuck is because pupils have a misconception which needs addressing by the teacher. At other times they cannot see the wood for the trees and cannot process the information that they are seeing. In this instance, encourage them to draw diagrams sorting out one phrase of information at a time looking for inconsistences in their thinking.

Ideas

Ideas come to one's brain in all sorts of ways. To form ideas pupils need mental space to think. Giving them time to work on their own is important even if they think it does not help. Be assured it does. At this stage be quiet. Don't clutter their brains by 'talking into' their mental space.

- As an aid use the 'Problem Solving in Mathematics' sheet from the Singapore mathematics session.
- Think about which branch of mathematics the problem may be about.
- Jot down what they know about his branch of mathematics.
- Draw a diagram to help visualise the links between the different pieces of information given in the question.

All of these things help to jog memories of strategies and approaches.

Having prior experience of the mathematical processes needed to solve problems increases the likelihood that pupils will succeed. As does being familiar with the context and language used in the question.

Pupils need the strategies in their memory banks. Unfortunately repeating a sheet of similar examples does not help pupils to have long term learning and understanding of mathematics. This strategy (of repeating similar examples) uses up a lot of working memory without deepening understanding. They need mathematical thinking / processing modelled. This should be interspersed with a variety of ways of approaching problems and thereby enabling pupils to test their thinking about when a process can /cannot be used.

Many ideas are transient and flitter across one's brain so being encouraged to record one word hints to help the pupils go back to ideas is useful.

Checking

Pupils should be encouraged to:

- go back and check that the answer they have found actually matches what they were asked to look for in the first place.
- make sure that the size of the number / pattern found makes sense.
- check the arithmetic / steps to ensure that each step of the process has been calculated properly.

Keeping a log: Reflection

Encouraging pupils to keep a thinking log of the ways in which they had to think to solve problems is a valuable meta–cognitive tool. The log acts as their own reference book which they can refer back to for techniques when they are 'stuck'. It also provides a delightful set of evidence about how they are moving forward in their thinking. According to Mason et al., those who fail to complete the log are missing out on both understanding themselves and on understanding the nature of their thinking. It takes time and effort in the first place and pupils may need help with key words. Try to avoid giving them specific examples as they will record them as their own thinking rather than reflecting for themselves. Is this process hard to teach and hard to learn? – yes. Is it worthwhile – YES!

It is helpful to record these ideas in the log:

- What thinking processes did you use to when trying out examples?
- What ideas came to mind when thinking - (the 'maybe's)?
- What were your feelings at different stages of the problem solving?

These notes need to be made at the time of solving the puzzle (not a lesson later or for homework) as pupils will not remember. To be useful the log needs to record their own thought processes and not those of their peers or yours!

For a much more in-depth look at this approach see Thinking Mathematically by Mason, Burton and Stacey

In summary

- Read carefully
- Find a method of specialising (draw diagrams, try out numbers, annotate etc.)
- Generalise (then re-specialise with new numbers to test)
- Check
- Reflect

Also remember

- When stuck
- When needing ideas

Problem Solving in Geometry

Research shows that many problem solving skills are not transferrable. One of the reasons for this is that the skill depends on the context. The more familiar a pupil is with the geometric concepts the easier it is for them to solve problems.

1. Know your geometric theorems.

2. Know your formulae related to shapes you can see (being able to say a formulae out loud whilst looking at the relevant shape helps fix it into the memory. It's no good saying 'πr^2' for the area of a circle without saying the whole – 'The area of a circle is πr^2').

3. Read and annotate the diagram or sketch / construct a diagram from the text. Reading the diagram involves describing it in your head using mathematical words like, circle, parallel lines, triangle, alternate angles etc. Looking at a diagram without hunting for patterns and descriptions inferred in it is a major cause of difficulty.

4. Think about which theorems can be applied – work your way through the different theorems and apply them across different parts of the diagram.

5. Stuck: then change to order in which you apply the theorems.

6. Stuck again: turn the diagram through 90° or 180°. A different perspective sometimes helps.

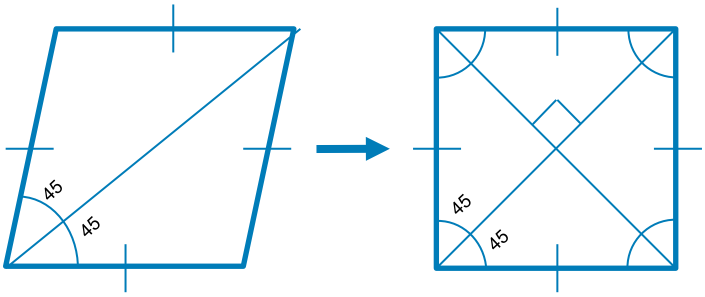
Example

A tiled floor is made up of rhombus shaped tiles. If one diagonal bisects its interior angle and forms an angle with a side of 45° and the area of the rhombus is 36cm^2, what is the length of one side of each tile?

What do I know about a rhombus?

1 - All sides are congruent (equal lengths).
2 - Opposite sides are parallel.
3 - The two diagonals are perpendicular to each other.
4 - Opposite interior angles are congruent (equal sizes).
5 - Any two consecutive interior angles are supplementary : they add up to 180°
6 - Area of any parallelogram is base x perpendicular height.

Draw a diagram



This is a square because all its sides are equal length. Each corner is a right angle (45° + 45°).

Area

$L^2 = 36cm^2$

$L = 6$ cm

Comments

a. Pupils need to be familiar with the theorems relating to geometric shapes to be able to sift through them so they can be applied.

b. Breaking down the question into ‘bits’ of information:

The original question:

A tiled floor is made up of rhombus shaped tiles. If one diagonal bisects its interior angle and forms an angle with a side of 45° and the area of the rhombus is $36cm^2$, what is the length of one side of each tile?

Break down the reading into phrases and apply each phrase to building / looking at / annotating the diagram. In this example, the information becomes:

- rhombus shape
- one diagonal bisects its interior angle
- and forms an angle with a side of 45°
- and the area of the rhombus is $36cm^2$,
- What is the length of one side of each tile?

Other methods and resources can be found at:

- This is a link to Craig Barton's advice and analysis of problem solving strategies for KS4. https://www.tes.com/teaching-resources/blog/tes-maths-rotw-multistage-problem-solving?utm_campaign=RES-1886&utm_content=maths-newsletter&utm_source=exact-target&utm_medium=email

- This is an entirely different way of looking at problem solving from the Mathematical Association of America. It contains short explanatory video clips. http://www.maa.org/math-competitions/teachers/curriculum-inspirations

- http://www.suffolkmaths.co.uk/pages/1ProblemSolving_KS3-4.htm

- STEM have put together a series of links to different problem solving sites. https://www.stem.org.uk/elibrary/list/25049/problem-solving

- Mathematical Problem Solving by Liu Yueh Mei and Soo Vei Li.
  This book explains and extends the bar method in more detail.

- Thinking Mathematically by John Mason

- Learning and Doing Mathematics: Using Polya's Problem-solving Methods for Learning and Teaching (Visions of Mathematics) by John Mason

- http://www.nuffieldfoundation.org/key-ideas-teaching-mathematics/introduction-0

- The NCETM have a section of their website which is devoted to developing problem solving in the mathematics department : https://www.ncetm.org.uk/resources/38609.

- For those interested in how problem solving may be assessed in the future:
  Problem solving in mathematics: realising the vision through better assessment June 2016 By ACME
  http://www.acme-uk.org/news/news-items-repository/2016/6/assessment-of-problem-solving-report

Appendix

How to approach a problem solving question

The problem:

Step 1: Understand the problem

What are you trying to find or do? Can you state the problem in your own words? What information can you obtain from the problem? What information, if any, is missing or not needed?

Step 2: Devise a plan

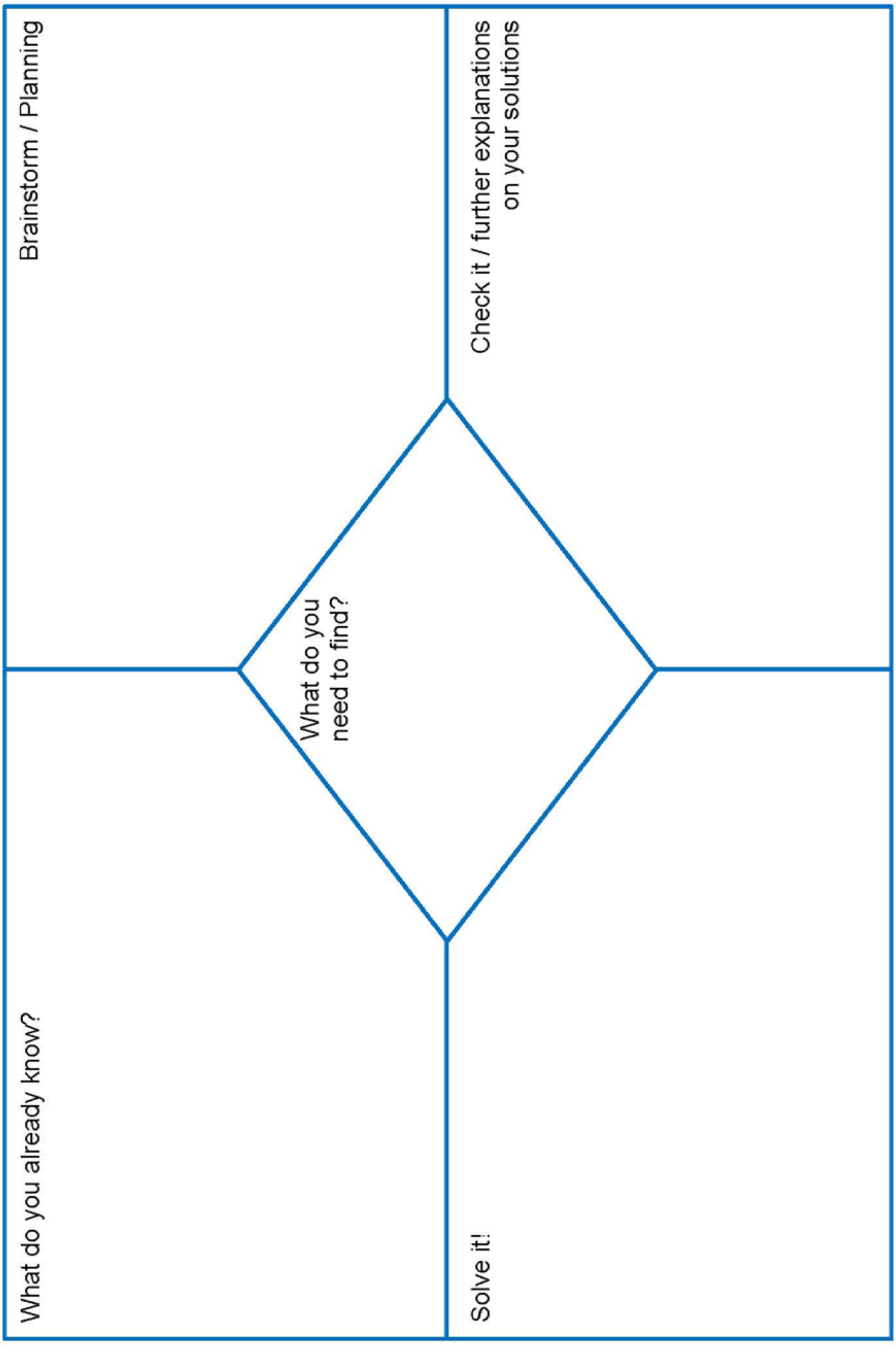
Look for a pattern. Use a flow diagram. Use x for the unknown. Consider similar problems. Draw a diagram. Make a table. Use simpler numbers. List the possibilities. Guess and check.

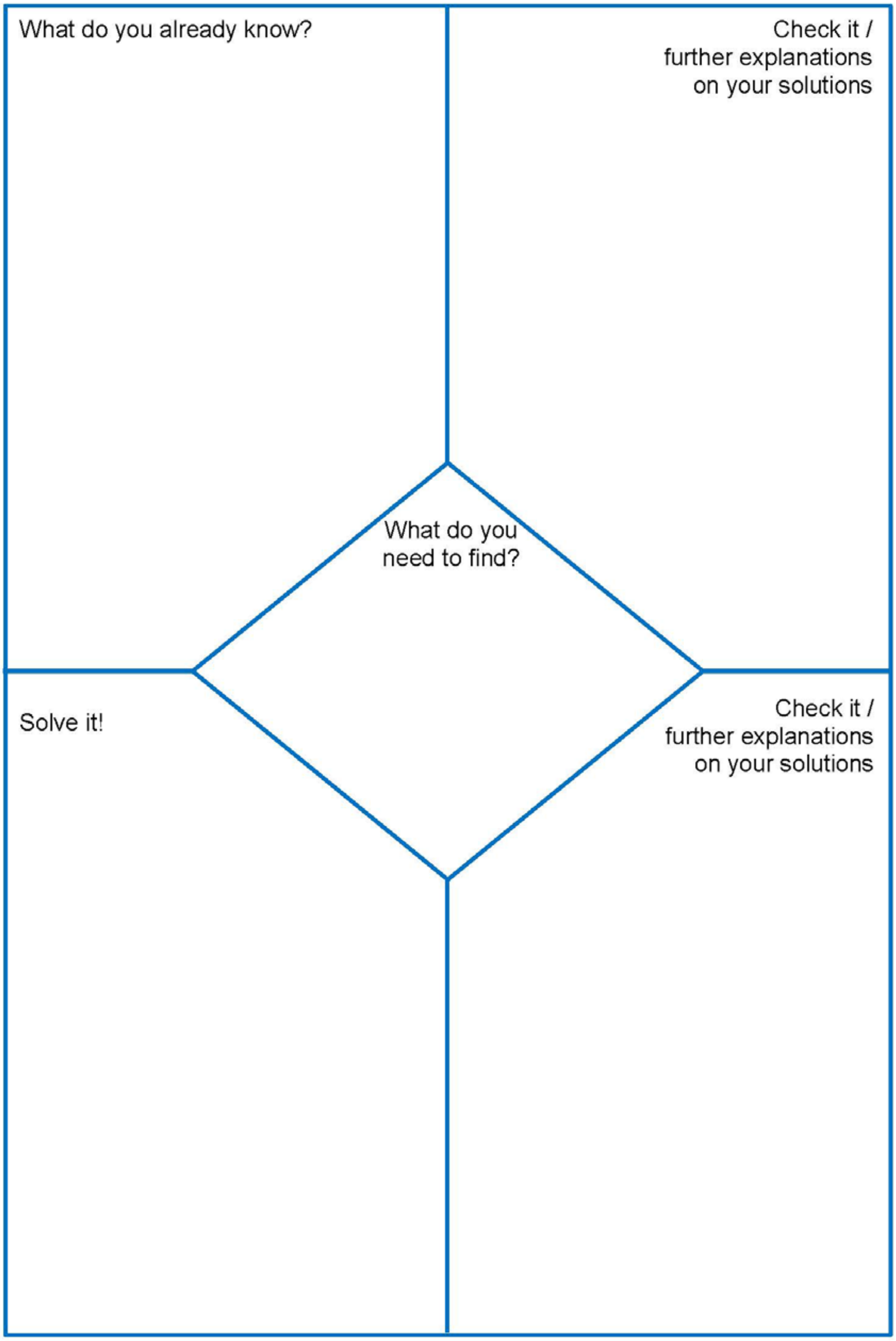
Step 3: Carry out the plan

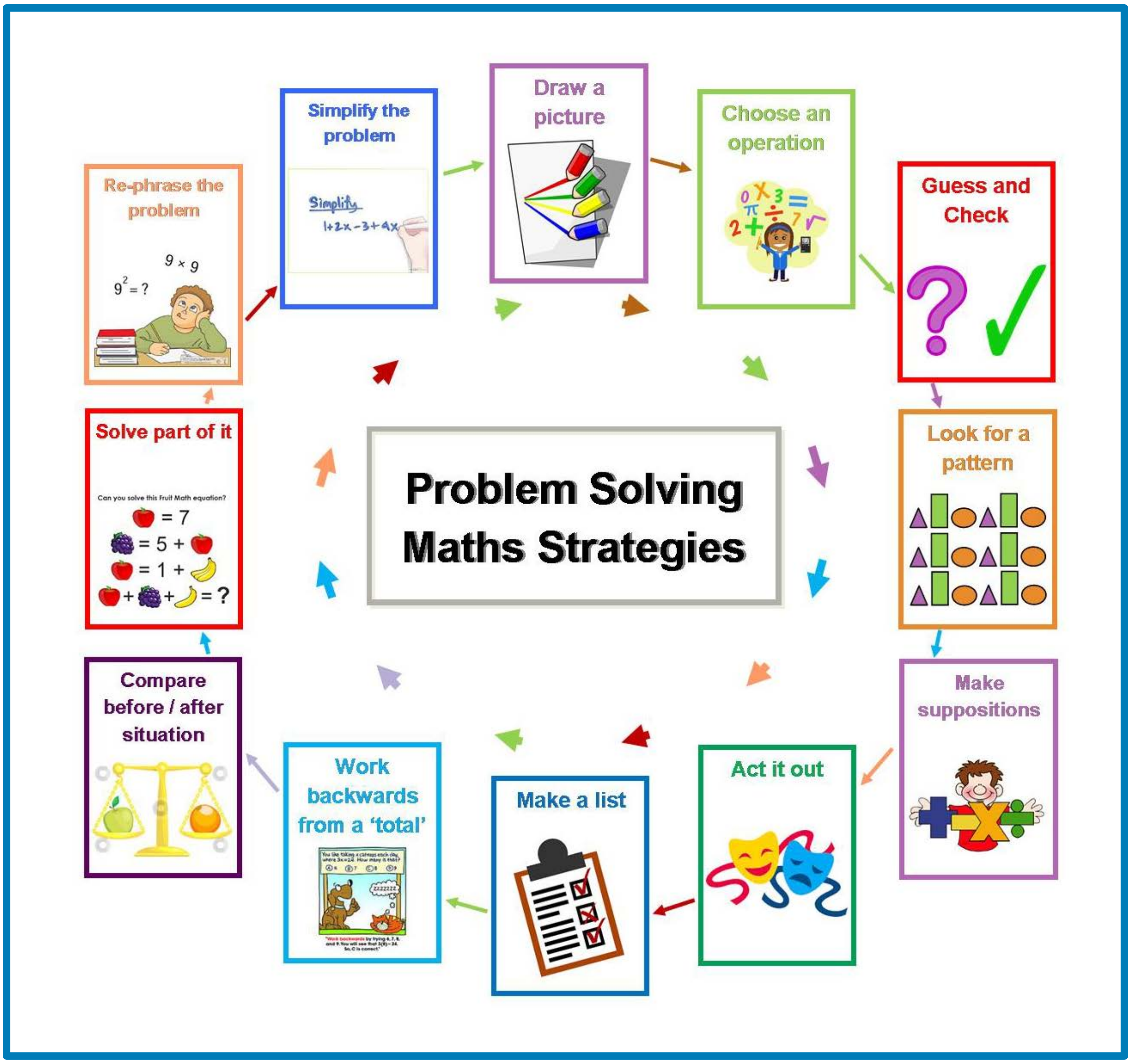
Keep an accurate record as you go. Check each step as you go. Are you keeping to your plan? Found a solution?

Step 4: Review your solution

Interpret your solution. Check your final answer. Check for other answers. Review your strategy. Does your solution seem reasonable?


Brainstorm / Planning
Check it / further explanations on your solutions
What do you need to find?
What do you already know?
Solve it!


What do you already know?
Check it / further explanations on your solutions
What do you need to find?
Solve it!
Check it / further explanations on your solutions



An addition to a Mathematics Teacher's Toolkit

Further copies of this booklet can be downloaded from:
[http:// boltonlearningtogether.bolton365.net/](http://boltonlearningtogether.bolton365.net/)

Bolton Learning Alliance

Bolton Council